

1. Równania o zmiennych rozdzielonych.

(a) $yy' + 4x = 0$

(b) $y' + 2xy^2, y(1) = 1.$

(c) $x(y^2 - 1) + y(x^2 - 1)y' = 0.$

(d) $2\sqrt{xy}' = \sqrt{1 - y^2}.$

(e) $y' = 1 + x + y + xy, y(-1) = 2.$

(f) $y' + 4y = y(e^{-x} + 4).$

(g) $(1 - x^2)y' = 2y.$

(h) $y' \sin x = y \ln y, y(\frac{\pi}{2}) = e.$

2. Jednorodne.

(a) $xy' = \sqrt{x^2 - y^2} + y.$

(b) $(x - y) + xy' = 0.$

(c) $xy' = y(\ln y - \ln x).$

(d) $xy' - y = x \operatorname{tg} \frac{y}{x}.$

(e) $(x^2 - y^2) + xyy' = 0.$

3. Liniowe

(a) $y' + y = \sin x.$

(b) $y' + 2xy = e^{-x^2}.$

(c) $xy' - 2y = x^3 \cos x.$

(d) $xy' - 2y = 4x^4.$

(e) $xy + e^x - xy' = 0.$

(f) $(2x + 1)y' = 4x + 2y.$

4. Znaleźć rozwiązanie szczególne metodą przewidywania.

(a) $y' + y = 2 \cos 2x + \sin 2x.$

(b) $y' + 2y = x^2 + 3x + 1.$

(c) $y' + y = e^{2x}(x + 1).$

5. Bernoulli.

(a) $xy' + y = y^2 \ln x, y(1) = 1.$

(b) $y' - 2y = 2\sqrt{y}e^x \ln x, y(1) = 0.$

- (c) $y' + 2xy = 2xy^2$.
 (d) $3xy^2y' - 2y^3 = x^3$.

Odpowiedzi.

1. (a) $y^2 + 4x^2 = C$.
 (b) $y(x) = -\frac{1}{x^2+C}$ oraz $y = 0$.
 (c) $y^2 = 1 + \frac{C}{1-x^2}$.
 (d) $y = \sin(C + \sqrt{x})$, $y = -1$.
 (e) $\ln(|1 + y|) = x + \frac{1}{2}x^2 + C$.
 (f) $y = Ce^{-e^{-x}}$.
 (g) $y = C\frac{1+x}{1-x}$.
 (h) $y = e^{\text{tg} \frac{x}{2}}$.
2. (a) $y = x \sin \ln |Cx|$, $y = x$, $y = -x$.
 (b) $y = x(C - \ln |x|)$.
 (c) $y = xe^{Cx+1}$.
 (d) $y = x \text{ arc tg } Cx$.
 (e) $|y| = |x|\sqrt{C - \ln x^2}$.
3. (a) $y = Ce^{-x} + \frac{1}{2}(\sin x - \cos x)$.
 (b) $y = (C + x)e^{-x^2}$.
 (c) $y = x^2(C + \sin x)$.
 (d) $y = Cx^2 + 2x^4$.
 (e) $y = (\ln |x| + C)e^x$.
 (f) $y = (\ln |2x + 1| + C)(2x + 1) + 1$.
4. (a) $y = \frac{3}{4} \cos 2x + \frac{1}{4} \sin 2x$.
 (b) $y = \frac{1}{2}x^2 + x$.
 (c) $y = e^{2x}(\frac{1}{3}x + \frac{2}{9})$.
5. (a) $y = \frac{1}{1+\ln x}$.
 (b) $y = (x \ln x - x + 1)^2 e^{2x}$.
 (c) $y = \frac{1}{1+Ce^{x^2}}$.
 (d) $y = \sqrt[3]{Cx^2 + x^3}$.